RESULTS OF LABORATORY PHOTOMETRIC MEASUREMENTS OF ARTIFICIAL PLANETS

O. I. Bugayenko, I. K. Koval', V. D. Krugov
A. V. Morozhenko, L. F. Slutskiy

ABSTRACT. Results are given of photoelectric measurements of brightness distribution and brightness-phase dependence for six spheres whose surfaces have different colors and roughness. These results (Tables 1 and 2) are applied to verify validity of some empirical formulas used earlier for determining optical parameters of the Martian surface.

Reference [1] hypothesized that the surface layer of a planet consists of /33 not too densely packed semitransparent particles of a size greatly in excess of that of the length of a light wave. This was done to explain the change that takes place in the phase curves of Martian brightness with wavelength $\tilde{\lambda}$. In this case the spectral shape of the phase curves can be explained quite well, qualitatively speaking, by the presence of the shadow effect in the particles, by the spectral behavior of the single scattering albedo, λ, and the degree of transparency, $\varkappa$, of the particles, as well as by the influence of the atmosphere in the short-wavelength region of the spectrum.

Reference [2] considered the following model of the surface layer structure:

1. the surface layer consists of small elements, spheres of radius r, not too tightly packed, so that light can penetrate the layer to some depth $h > 2r$ from any direction, and will be scattered and absorbed by these particles;

2. particle dimensions exceed the length of the light wave, $\tilde{\lambda}$, such that a well-defined shadow forms behind each particle;

3. the particles are transparent. The degree of transparency is determined by the magnitude $\varkappa$, which indicates how much of the luminous flux entering the contour of the particle leaves a parallel beam absorbed and scattered to the side;

4. diffraction of the light by the particles can be ignored.

Reference [2] then used this model to obtain an approximate formula for /34 the intensity, $I(\eta, \zeta, \alpha)$ of the light diffusely reflected by the surface

(the reflection law)

$$I(\eta, \zeta, \alpha) = I_1(\eta, \zeta, \alpha) + \Delta I(\eta, \zeta, \alpha), \tag{1}$$

where

$$I_1(\eta, \zeta, \alpha) = \frac{\lambda}{4}\frac{\zeta}{\eta + \zeta}\chi(\pi - \alpha)\exp\left[\frac{\varkappa^2}{\pi}\left(\frac{g}{2}\right)^{3/2}\operatorname{ctg}\frac{\alpha}{2}\right] \times J(y, \varkappa), \tag{2}$$

$$\Delta I(\eta, \zeta, \alpha) = \frac{\lambda}{4}\frac{\zeta}{\eta + \zeta}\left\{\varphi_0^0(\eta)\,\varphi_0^0(\zeta) - 1 - x_1\left[\varphi_1^0(\eta)\,\varphi_1^0(\zeta) - \frac{\eta\zeta - \cos\alpha}{\sqrt{(1-\eta^2)(1-\zeta^2)}}\,\varphi_1^1(\eta)\,\varphi_1^1(\zeta) - \cos\alpha\right]\right\}, \tag{3}$$

where

$\chi(\pi - \alpha)$ is the particle scattering indicatrix;

α is the phase angle;

ζ, η are the cosines of the angles of incidence and reflection of light, respectively;

g is a factor for particle packing density in a layer

$$y(\eta, \zeta, \alpha) = \frac{3\varkappa}{4}\left(\frac{g}{2}\right)^{3/2}\left(\frac{1}{\eta} + \frac{1}{\zeta}\right)\sqrt{1 + \eta\zeta\operatorname{cosec}^2\frac{\alpha}{2}}. \tag{4}$$

Numerical values of the function $j(y, \varkappa)$ are given in reference [2], and precise values of the Ambartsumyan function, $\varphi_i^k(\eta)$, when $|x_1| \leq 1$, can be taken from the tables compiled by I. N. Minin et al. [3]. The Ambartsumyan function scattering indicatrix can be calculated for the other values of the elongation, x_1, by using the approximate formulas derived by E. G. Yanovitskiy [4].

As will be seen from Eqs. (2) and (3), the optical model of the surface layer can be determined by the assignment of parameters g, λ, $\varkappa$, and by the scattering indicatrix $\chi(\pi - \alpha)$, which can be found by comparing the observed and calculated intensity values. At the same time, data on planetary disk brightness distribution can be used, as can data on the integral brightness phase relationship. The maximum phase angle for Mars is, as we know, 47°, so the observed phase curve for the brightness of this planet cannot be fully used to determine the scattering indicatrix, $\chi(\pi - \alpha)$. However, the parameter λ can be found [5] from the data on the Martian disk brightness distribution.

Reference [5] proposed an interpolation formula for determining a planet's phase function, $\Phi(\alpha)$, for $0 \leq \alpha \leq 180°$, if this function is known within a limited range of phase angles $0 \leq \alpha \leq \alpha_0$ ($\alpha_0 = 47°$ for Mars). But if the phase

function is completely known, the scattering indicatrix, $\chi(\pi - \alpha)$, and all the optical parameters can be determined quite readily.

We made laboratory measurements of the phase relationship and disk brightness distribution for artificial planets with different surface colors and roughnesses in December 1969, in order to verify the validity of the interpolation formula in reference [5]. The task assigned the experiment also included (1) a check on the degree of agreement between λ values found from disk brightness distribution on the one hand, and with respect to the planet brightness phase relationship on the other, and (2) study of the relationship between planetary disk brightness distribution and λ. It was of interest, in passing, to check the validity of Russell's empirical formula [6].

Six spheres, 60 mm in diameter, covered with a layer of powder as follows, /35 were used in the experiment: (I) crocus (mean grain size d = 200 microns); (II) brown hematite (d = 100 microns); (III) brown hematite + sand (d = 150 microns); (IV) chrome carborundum (d = 100 microns); (V) chrome carborundum (d = 250 microns); and (VI) chrome carborundum (d = 400 microns). The coatings were applied to spheres I, II, and III by the laboratory at the Belgorod Experimental Industrial Abrasive Plant. The powders were applied in a casein binder in several layers such that the backing was not translucent. A similar technology was used by the laboratory at the Main Astronomical Observatory of the Academy of Sciences of the Ukrainian SSR to coat spheres IV, V, and VI. Figure 1 is a photograph of the side-lighted surfaces of the spheres, magnified 6 times.

The equipment described in reference [7] was used to measure the integral brightness-phase relationship for the spheres, as well as the brightness distribution along the intensity equator.

The surfaces of the spheres were illuminated by a parallel beam of light from a ribbon filament lamp supplied by a stabilized power source. Steps were taken to do away with extraneous illumination. Phase relationships were measured between phase angles 0.7° - 170° in 10° increments. Brightness distribution was measured with a diaphragm with a diameter of $0.2R_s$, with a spacing of approximately $0.1R_s$ for phase $\alpha = 0.7°$, and for sphere III for $\alpha = 10$ and 37.5° as well. All measurements of spheres I, II, and III were made in four regions

of the spectrum defined by interference filters. Spheres IV, V, and VI were measured in one region of the spectrum (596 millimicrons). Measurements of a white screen with the following values of reflectivity, ρ, were made to determine surface reflectivity:

$\tilde{\lambda}$, millimicrons	416	503	596	750	б/ф
ρ	0.74	0.84	0.89	0.90	0.86

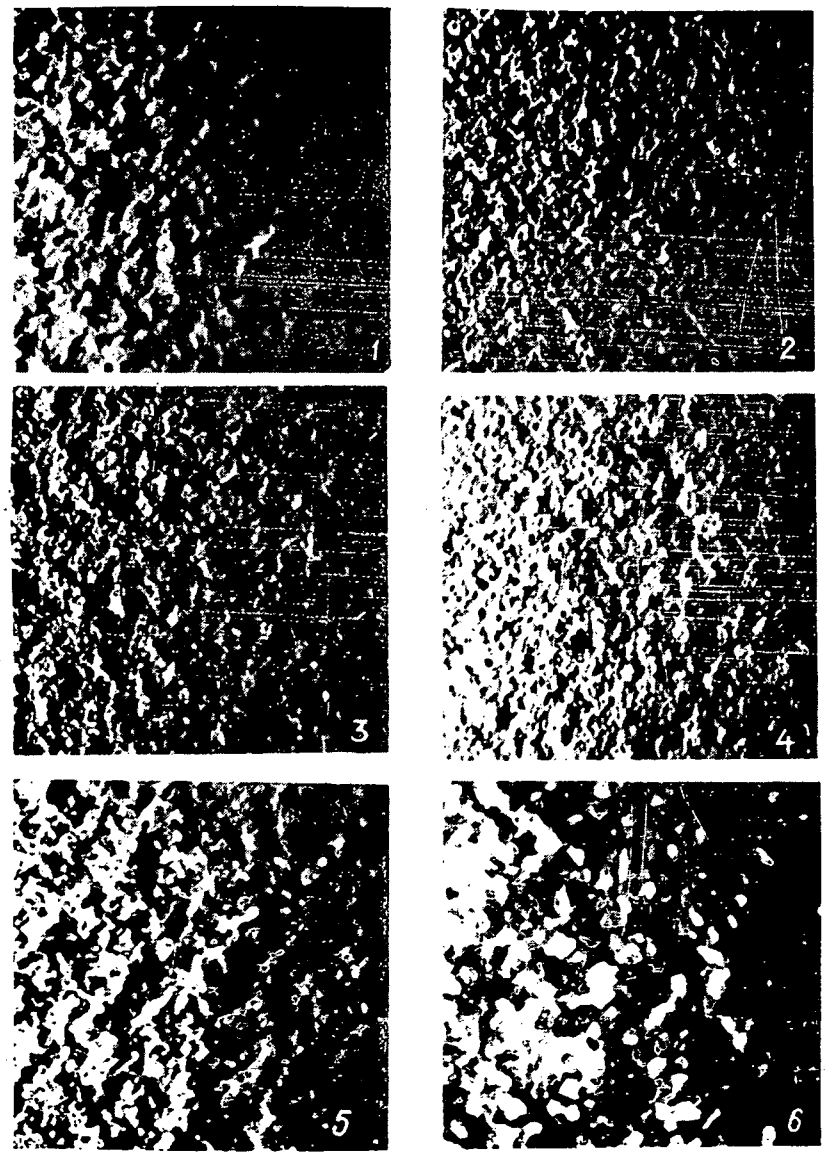

Figure 1. Views of surface of spheres (I-VI) magnfied 6 times.

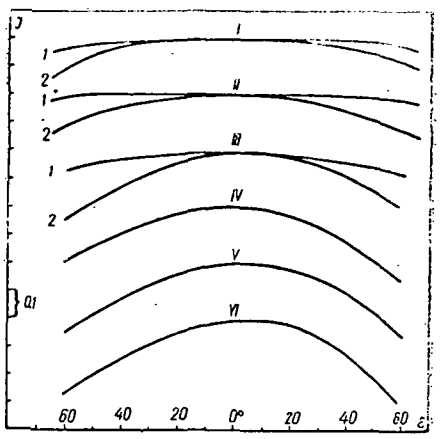

Figure 2. Distribution of brightness along the intensity equator for spheres with $\alpha = 0.7°$. For spheres I-III 1 - 416 millimicrons, 2 - 750 millimicrons; for spheres IV-VI - 596 millimicrons.

The results of the measurement of brightness distribution along the intensity equator are listed in Tables 1 and 2. Reflectivity, ρ, for each sphere is listed at the end of the table. Figure 2 shows the brightness distribution for $\alpha = 0.7°$ for the spectral extremes (I, II, III) and in one spectral region for spheres IV, V, and VI. Table 3 lists the results of the measurements of the phase-brightness relationship for the spheres for extreme wavelength values. The values of the geometric albedo, A_g, and phase integral, q, are listed at the end of the table.

As we see from Figure 2 and Table 3, sphere disk brightness distribution, and the value of the phase functions and the phase integral depend on the wavelength, $\tilde{\lambda}$ (that is, on surface reflectivity). The reflectivity of the models studied decreased with decrease in the limb opacity. L. A. Akimov and N. P. Barabashov [8, 9] obtained similar results from measuring a great many laboratory models.

This peculiarity in the reflection law is readily explained (within the context of the model adopted for the structure of a ploughed up surface) by the change in the parameters λ and $\varkappa$ with wavelength in Eqs. (1) - (3), which provide a quantitative description of the reflection law for a ploughed up surface. /36

Let us now move on to the indirect purpose of the experiment, that of ascertaining the validity of the procedure used [5] to determine the phase function and the parameter λ. One is readily persuaded that the product /37

$\chi(\pi-\alpha)\exp\left[\frac{\varkappa^2}{\pi}\left(\frac{\sigma}{2}\right)^3 \operatorname{ctg}\frac{\alpha}{2}\right] J(y,\varkappa)$ has practically no dependence on η and ζ when α is small ($\alpha \leq 30°$). In this case change in the function $\frac{\eta-\zeta}{\zeta} I(\eta,\zeta,\alpha)$ with change in η and ζ is almost entirely attributable to multiple particle scattering of the surface layer, and this permits the determination of the value of the

parameter λ in terms of the magnitude $\Delta I(\eta, \zeta, \alpha)$ quite confidently.

TABLE 1. BRIGHTNESS DISTRIBUTION ALONG THE INTENSITY EQUATOR OF ARTIFICIAL PLANETS (α = 0.7°).

ε	I				II				IV	V	VI
	$\tilde{\lambda}$, millimicron										
	416	503	596	750	416	503	596	750	596	596	596
65°	0.95	0.97	0.94	0.86	0.98	0.98	0.95	0.86	0.77	0.69	0.67
60	0.97	0.98	0.96	0.91	0.99	0.99	0.97	0.90	0.80	0.75	0.72
50	0.98	0.99	0.98	0.95	1.00	1.00	0.98	0.93	0.86	0.82	0.80
40	0.99	1.00	0.99	0.97	1.00	1.00	0.99	0.95	0.91	0.88	0.85
30	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.96	0.94	0.94	0.89
25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.97	0.96	0.92
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.99	0.98	0.95
15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	0.99	0.97
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
15	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99
20	1.00	1.00	1.00	0.99	1.00	1.00	0.98	0.98	0.98	0.98	0.98
25	1.00	1.00	0.99	0.98	1.00	1.00	0.97	0.97	0.96	0.97	0.96
30	1.00	1.00	0.98	0.97	1.00	1.00	0.96	0.95	0.93	0.95	0.92
40	1.00	1.00	0.97	0.96	1.00	1.00	0.95	0.93	0.88	0.90	0.86
50	1.00	1.00	0.95	0.95	1.00	1.00	0.94	0.90	0.82	0.83	0.79
60	0.99	0.99	0.93	0.93	0.99	0.99	0.93	0.88	0.74	0.75	0.70
65	0.97	0.97	0.90	0.90	0.98	0.98	0.91	0.85	0.69	0.69	0.64
ϱ	0.039	0.042	0.117	0.190	0.035	0.087	0.256	0.352	0.453	0.324	0.255

TABLE 2. BRIGHTNESS DISTRIBUTION ALONG THE INTENSITY EQUATOR OF ARTIFICIAL PLANETS

ε	III, α=0°.7				III, α=10°				III, α=37°.5			
	$\tilde{\lambda}$, millimicron											
	416	503	596	750	416	503	596	750	416	503	596	750
60	0.94	0.93	0.85	0.78	0.98	1.00	0.94	0.86	1.27	1.24	1.22	1.16
50	0.95	0.95	0.88	0.82	1.00	1.01	0.96	0.88	1.24	1.22	1.19	1.16
40	0.96	0.96	0.91	0.86	1.00	1.01	0.96	0.92	1.21	1.20	1.16	1.12
30	0.97	0.97	0.94	0.90	1.00	1.01	0.98	0.96	1.15	1.16	1.13	1.12
25	0.98	0.98	0.97	0.94	1.00	1.01	0.98	0.98	1.12	1.13	1.12	1.11
20	0.99	0.99	0.98	0.96	1.00	1.00	1.00	1.00	1.09	1.10	1.11	1.09
15	0.99	1.00	0.99	0.98	1.00	1.00	1.00	1.00	1.06	1.08	1.08	1.06
10	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.03	1.06	1.05	1.05
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.03	1.02	1.03
0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
5	1.00	1.00	0.99	1.00	0.98	0.98	0.98	0.98	0.96	0.95	0.94	0.96
10	0.99	1.00	1.99	0.99	0.96	0.97	0.98	0.96	0.90	0.91	0.88	0.90
15	0.99	0.99	0.98	0.99	0.94	0.96	0.96	0.94	0.84	0.87	0.83	0.83
20	0.98	0.98	0.98	0.98	0.92	0.94	0.94	0.92	0.78	0.83	0.77	0.80
25	0.98	0.98	0.97	0.96	0.90	0.93	0.92	0.88	0.72	0.77	0.72	0.70
30	0.97	0.97	0.96	0.94	0.88	0.92	0.90	0.84	0.63	0.71	0.61	0.61
40	0.96	0.96	0.94	0.91	0.86	0.89	0.86	0.78	0.54	0.63	0.47	0.41
50	0.94	0.95	0.91	0.86	0.84	0.86	0.80	0.73				
60	0.93	0.94	0.87	0.81	0.80	0.80	0.74	0.65				
ϱ	0.051	0.104	0.263	0.340								

TABLE 3. SPHERE PHASE-BRIGHTNESS RELATIONSHIP

α	I		II		III		IV	V	VI
	λ, millimicron								
	416	750	416	750	416	750	596	596	596
0°.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	0.73	0.78	0.70	0.73	0.70	0.74	0.87	0.91	0.93
20	0.60	0.64	0.53	0.64	0.60	0.65	0.82	0.86	0.88
30	0.50	0.54	0.41	0.55	0.48	0.56	0.73	0.76	0.79
40	0.41	0.45	0.35	0.48	0.40	0.49	0.65	0.69	0.70
50	0.34	0.38	0.28	0.40	0.33	0.42	0.57	0.60	0.62
60	0.29	0.32	0.22	0.33	0.27	0.35	0.48	0.53	0.54
70	0.25	0.27	0.18	0.27	0.23	0.29	0.41	0.45	0.47
80	0.20	0.22	0.14	0.22	0.18	0.23	0.34	0.37	0.39
90	0.16	0.18	0.11	0.17	0.15	0.18	0.26	0.30	0.31
100	0.13	0.14	0.08	0.12	0.12	0.13	0.19	0.24	0.25
110	0.10	0.11	0.06	0.09	0.09	0.10	0.14	0.19	0.20
120	0.08	0.09	0.05	0.06	0.07	0.08	0.10	0.15	0.16
130	0.07	0.08	0.04	0.05	0.06	0.06	0.07	0.10	0.13
140	0.06	0.07	0.03	0.04	0.05	0.04	0.04	0.07	0.10
150	0.04	0.05	0.02	0.03	0.04	0.03	0.03	0.05	0.08
160	0.03	0.04	0.02	0.02	0.03	0.02	0.02	0.03	0.06
170	0.02	0.03	0.01	0.01	0.02	0.01	0.01	0.02	0.04
A_g	0.028	0.194	0.024	0.356	0.035	0.342	0.388	0.270	0.204
q	0.81	0.87	0.62	0.82	0.75	0.88	1.22	1.37	1.43

Let us find the value of λ, using the data on brightness distribution for spheres II and III ($\tilde{\lambda}$ = 750 millimicrons - Table 1 and 2). The size of the diaphragm for measuring the brightness distribution was $\sim 0.2R_s$, so the maximum reflection angle $\varepsilon \approx 65°$ ($\alpha = 0.7°$) is the mean for the interval $52° \leq \varepsilon \leq 90°$. Now

$$\Delta\bar{I}(\bar{\varepsilon}) = \int_{\varepsilon_1}^{\varepsilon_2} \Delta I(\varepsilon)\, d\varepsilon. \tag{5}$$

Making all the necessary calculations we obtain $\lambda = 0.76$ for sphere III, and $\lambda = 0.68$ for sphere II.

On the other hand, if the brightness phase curve is known, λ can be determined from

$$2\int_0^{\pi} \frac{h(\alpha)}{G(\alpha, g, \varkappa)} \sin\alpha d\alpha = \lambda + 2\int_0^{\pi} \frac{\Delta G(\alpha, \lambda, X_1)}{G(\alpha, g, \varkappa)} \sin\alpha d\alpha, \tag{6}$$

where

$$h(\alpha) = \frac{\pi}{2} A_g(\alpha); \tag{7}$$

$A_g(\alpha)$ is the apparent geometric albedo. /38

$$G(\alpha, g, \varkappa) = \exp\left[\frac{\varkappa^2}{\pi}\left(\frac{g}{2}\right)^{3/2}\operatorname{ctg}\frac{\alpha}{2}\right]\int\limits_{\alpha-\frac{\pi}{2}}^{\pi/2}\frac{\eta\zeta}{\eta+\zeta}\,d\omega\int\limits_{0}^{\pi/2}J(y,\varkappa)\cos\psi d\psi; \tag{8}$$

$$\Delta G(\alpha, \lambda, X_1) = \int\limits_{\alpha-\frac{\pi}{2}}^{\pi/2}\eta d\omega\int\limits_{0}^{\pi/2}\Delta I(\eta, \zeta, \alpha)\cos\psi d\psi; \tag{9}$$

$$\eta = \cos\omega\cos\psi,\ \zeta = \cos(\alpha-\omega)\cos\psi; \tag{10}$$

ω and ψ are the planetocentric longitude and latitude of a point on the planet's surface. The numerical values of the functions $G(\alpha, g, \varkappa)$ and $\Delta G(\alpha, \lambda, x_1)$ are listed in Tables 4 and 5. Approximation formulas [4] for the Ambartsumyan function φ_i^k were used to calculate the $\Delta G(\alpha, \lambda, x_1)$.

Analysis reveals that the λ values obtained using Eq. (6) are little dependent on x_1, g, and $\varkappa$. Their roughly approximate values therefore can be taken in this case. Eq. (6) yielded $\tilde{\lambda} = 0.74$ and $\lambda = 0.68$, respectively, for spheres III and II, and these values agree well with the λ values found from the disk brightness distribution. /47

The maximum phase angle when observing Mars is 47°. But there are cases when the requirement is for knowledge of the planet's spherical albedo, $A_s = A_g q$, or its brightness for any phase angle value $0° \leq \alpha \leq 180°$. Russell [6] has suggested an empirical formula in which $A_s = aA_g\Phi(50°)$, where $a \approx 2.2$, for finding the spherical albedo. Let us check the validity of this expression, using the results of the measurements made of the spheres. In our case, according to the calculations, $a = 2.29 \pm 0.11$.

The following formula has been suggested for use in finding the Martian phase function for $\alpha > 47°$ [5] /48

$$\Phi(\alpha) = a\alpha^3 + b\alpha^2 + c\alpha + d, \tag{11}$$

in which the constants a, b, c, and d are found from the following family of equations

$$\Phi(\alpha_0) = a\alpha_0^3 + b\alpha_0^2 + c\alpha_0 + d,$$
$$\left.\frac{d\Phi}{d\alpha}\right|_{\alpha=\alpha_0} = 3a\alpha_0^2 + 2b\alpha_0 + c,$$
$$\Phi(180°) = 0, \qquad (12)$$
$$\left.\frac{d\Phi}{d\alpha}\right|_{\alpha=180°} = 0.$$

TABLE 4.

$G(\alpha, g, \varkappa) \cdot 10^3$

α	g								
	0.1	.2	.3	.4	.5	.6	.7	.8	.9
					$\varkappa=1$				
0°	1571	1571	1571	1571	1571	1571	1571	1571	1571
1	1017	1212	1321	1385	1425	1452	1471	1485	1497
2	919	1077	1193	1273	1328	1367	1396	1417	1434
3	878	1004	1112	1194	1255	1300	1335	1361	1382
4	854	959	1056	1135	1198	1246	1283	1313	1336
5	839	927	1014	1089	1151	1200	1239	1271	1296
10	797	846	900	953	1002	1045	1083	1115	1143
	768	801	839	879	916	951	982	1010	1035
20	741	765	794	823	853	881	906	930	951
25	713	732	754	777	801	823	844	864	881
30	684	699	716	735	754	772	789	805	819
35	653	665	680	695	710	725	738	751	763
40	622	632	643	656	668	680	691	701	710
45	589	598	607	617	627	637	646	654	661
50	556	563	571	580	588	596	602	609	614
60	490	494	500	505	511	516	519	523	525
70	423	427	430	434	437	440	441	443	443
80	359	361	363	365	367	368	368	368	368
90	297	298	300	301	301	301	300	299	298
100	239	240	240	241	240	240	238	237	234
110	186	186	186	186	185	184	182	180	178
120	138	139	138	138	137	135	133	131	128
130	97.2	97.2	96.7	96.0	94.7	93.2	91.2	89.0	86.4
140	62.8	62.6	62.1	61.2	60.0	58.6	56.7	54.7	52.5
150	35.5	35.3	34.8	34.8	32.9	31.6	30.1	28.7	27.8
160	15.8	15.6	15.2	14.5	13.7	13.7	11.8	10.8	9.8
170	3.9	9.8	3.5	3.1	2.7	2.3	1.9	1.7	1.4
					$\varkappa=0.9$				
0°	1428	1428	1428	1428	1428	1428	1428	1428	1428
1	976	1139	1230	1282	1314	1336	1351	1362	1370
2	894	1025	1123	1189	1235	1367	1291	1308	1321
3	860	964	1055	1124	1175	1212	1241	1263	1279
4	841	927	1007	1074	1126	1167	1198	1223	1242
5	828	900	972	1035	1087	1128	1162	1188	1209
10	791	831	876	920	960	997	1029	1056	1079
15	765	792	822	855	886	915	942	966	986
20	738	758	782	806	830	854	875	895	913
25	711	726	744	764	783	802	819	836	850
30	682	694	709	724	740	755	769	783	795

TABLE 4 CONTINUED

a	$\varkappa$								
	0.1	.2	.3	.4	.5	.6	.7	.8	.9
35°	652	662	674	686	699	711	722	735	743
40	620	629	638	649	659	669	678	687	695
45	588	595	603	611	620	628	636	642	648
50	556	561	568	575	582	588	594	599	604
60	489	495	498	502	507	511	514	517	519
70	423	426	429	432	434	437	438	440	440
80	538	360	362	364	365	366	367	367	366
90	297	298	299	300	300	301	300	299	298
100	239	240	240	240	240	240	239	237	236
110	186	186	186	186	186	185	183	182	180
120	138	139	138	138	137	136	134	133	130
130	97.2	97.2	96.9	96.3	95.3	94.1	92.5	90.7	88.6
140	62.8	62.6	62.2	61.6	60.6	59.4	58.0	56.3	54.3
150	35.5	35.4	34.9	34.3	33.5	32.4	31.2	29.9	28.4
160	15.9	15.7	15.3	14.8	14.1	13.4	12.6	11.6	10.7
170	3.95	3.81	3.58	3.28	2.91	2.55	2.21	1.91	1.64
	$\varkappa=0.8$								
0°	1309	1309	1309	1309	1309	1309	1309	1309	1309
1	938	1073	1149	1192	1219	1236	1249	1257	1264
2	872	979	1060	1116	1154	1181	1200	1214	1225
3	845	929	1003	1061	1103	1135	1159	1177	1191
4	829	898	964	1019	1063	1097	1123	1144	1160
5	818	876	935	987	1030	1065	1092	1115	1132
10	786	818	854	890	923	953	980	1003	1023
15	761	783	808	834	859	883	905	925	943
20	736	752	770	790	810	829	847	864	879
25	709	721	736	751	767	783	797	811	823
30	681	690	702	714	727	740	751	763	773
35	651	659	668	678	688	699	708	717	725
40	619	626	634	642	650	659	667	674	680
45	587	593	599	606	613	620	626	632	637
50	554	559	565	570	576	581	586	591	595
60	489	492	495	499	503	506	509	512	514
70	423	425	427	430	432	434	436	437	438
80	358	360	361	363	364	365	366	366	366
90	297	298	298	299	300	300	300	299	298
100	239	239	240	240	240	240	239	238	237
110	186	186	186	186	186	185	184	183	181
120	138	139	138	138	138	137	136	134	132
130	97.2	97.2	97.0	97.5	95.8	94.9	93.7	92.3	90.7
140	62.8	62.6	62.3	61.9	61.1	60.3	59.2	57.7	56.4
150	35.5	35.5	35.0	34.7	34.0	33.1	32.1	31.0	30.0
160	15.9	15.8	15.4	15.0	14.5	13.9	13.2	12.4	11.6
170	3.96	3.85	3.65	3.42	3.12	2.81	2.50	2.27	2.02
	$\varkappa=0.7$								
0°	1208	1208	1208	1208	1208	1208	1208	1208	1208
1	905	1015	1078	1114	1136	1150	1160	1167	1172
2	852	937	1004	1050	1082	1104	1120	1132	1141
3	831	897	957	1004	1040	1066	1086	1101	1113
4	818	872	925	970	1006	1035	1057	1074	1088
5	809	855	901	943	978	1007	1031	1050	1064
10	782	806	834	863	890	914	936	956	972
15	758	775	794	815	835	855	873	889	904
20	734	746	761	776	792	807	822	835	848
25	707	717	728	740	753	765	777	788	790
30	679	687	696	706	716	726	735	744	752
35	650	656	663	671	679	687	695	702	709
40	619	624	630	636	643	650	656	662	667
45	587	591	596	601	607	612	617	622	627
50	554	558	562	566	571	575	579	583	587
60	488	491	494	497	499	502	505	507	509
70	422	424	426	428	430	431	433	434	435
80	358	359	360	362	363	364	364	364	364
90	296	297	298	299	299	299	299	299	299

TABLE 4 CONTINUED

α	ξ								
	0.1	.2	.3	.4	.5	.6	.7	.8	.9
100°	239	239	240	240	240	240	239	239	238
110	186	186	186	186	186	186	185	184	183
120	138	138	138	138	138	137	136	135	134
130	97.2	97.2	97.1	96.8	96.3	95.6	94.7	93.7	92.5
140	62.8	62.7	62.5	62.2	61.6	61.0	60.1	59.1	58.1
150	35.5	35.5	35.2	35.0	34.4	33.8	33.1	32.2	31.4
160	15.9	15.8	15.6	15.3	14.9	14.4	13.9	13.3	12.6
170	3.97	3.89	3.75	3.56	3.33	3.07	2.80	2.63	2.39
				$\varkappa=0.6$					
0°	1122	1122	1122	1122	1122	1122	1122	1122	1122
1	875	963	1015	1045	1063	1075	1083	1089	1093
2	835	900	954	992	1018	1037	1050	1060	1068
3	819	869	916	954	983	1005	1022	1035	1045
4	808	849	890	926	955	979	997	1012	1023
5	801	836	871	904	933	956	976	991	1004
10	778	796	817	839	860	879	897	913	926
15	756	768	783	798	814	829	843	856	868
20	732	741	752	764	776	788	799	810	820
25	706	713	721	731	740	750	759	768	776
30	678	684	690	698	705	713	721	728	735
35	649	653	659	665	671	677	683	689	695
40	618	622	626	631	636	641	646	651	656
45	586	589	593	597	601	605	609	613	617
50	554	556	559	563	566	570	573	576	579
60	488	490	492	494	496	499	501	503	504
70	422	423	425	426	428	429	430	431	432
80	358	359	360	361	361	362	363	363	363
90	296	297	297	298	298	299	299	299	299
100	239	239	239	240	240	240	239	239	238
110	186	186	186	186	186	186	185	185	184
120	138	138	138	138	138	138	137	136	136
130	97.2	97.2	97.1	96.9	96.6	96.1	95.5	94.8	94.0
140	62.8	62.7	62.6	62.4	62.0	61.5	61.0	60.3	59.5
150	35.6	35.5	35.4	35.1	34.8	34.3	33.8	33.2	32.5
160	15.9	15.8	15.7	15.5	15.2	14.8	14.4	14.0	13.5
170	3.97	3.82	3.68	3.51	3.31	3.10	2.88	2.68	2.53
				$\varkappa=0.5$					
0°	1047	1047	1047	1047	1047	1047	1047	1047	1047
1	850	916	958	983	999	1009	1015	1020	1024
2	820	868	909	940	961	977	968	997	1003
3	808	844	879	909	933	951	965	975	984
4	800	830	860	887	910	929	944	956	966
5	795	819	845	870	892	911	926	939	950
10	774	787	802	818	834	849	862	874	885
15	754	762	773	784	795	806	817	827	837
20	730	737	744	753	762	770	779	787	795
25	705	710	716	722	729	736	743	750	756
30	677	681	686	690	697	702	708	712	718
35	648	651	655	659	664	668	673	677	682
40	617	620	623	627	630	634	638	641	645
45	586	588	590	593	596	599	602	605	608
50	553	555	557	560	562	565	567	570	572
60	488	489	490	492	494	495	497	498	500
70	422	423	424	425	426	427	428	429	430
80	358	358	358	359	360	360	361	361	362
90	296	297	297	298	297	298	298	298	298
100	239	239	239	239	239	239	239	239	239
110	186	186	186	186	186	186	186	185	185
120	138	138	138	138	138	138	138	137	137
130	97.2	97.2	97.2	97.0	96.9	96.6	96.2	95.7	95.2
140	62.8	62.8	62.7	62.5	62.3	62.0	61.6	61.2	60.6
150	35.6	35.5	35.4	35.3	35.1	34.8	34.4	34.0	33.6
160	15.9	15.8	15.8	15.6	15.4	15.2	14.9	14.6	14.3
170	3.98	3.94	3.88	3.78	3.66	3.52	3.36	3.20	3.03

TABLE 4 CONTINUED

α	g								
	0.1	.2	.3	.4	.5	.6	.7	.8	.9
					$\varkappa=0.4$				
0°	982	982	982	982	982	982	982	982	982
1	828	876	908	929	941	950	956	960	963
2	808	840	870	893	911	924	933	940	945
3	799	824	848	870	888	902	914	922	929
4	794	813	834	853	870	885	897	907	915
5	790	806	823	841	857	871	883	893	902
10	772	780	790	801	811	822	832	841	849
15	752	757	764	772	779	787	795	802	809
20	729	733	738	744	750	756	762	767	773
25	704	707	711	715	720	725	729	734	739
30	676	679	682	685	689	693	697	701	704
35	647	649	652	655	658	661	664	667	670
40	617	618	620	623	625	628	630	633	635
45	585	587	588	590	592	594	596	599	601
50	553	554	556	557	559	561	562	564	566
60	487	488	489	490	491	493	494	495	496
70	422	422	423	424	424	425	426	427	427
80	357	358	358	359	359	360	360	361	361
90	296	296	297	297	297	297	298	298	298
100	238	239	239	239	239	239	239	239	239
110	186	186	186	186	186	186	186	186	185
120	138	138	138	138	138	138	138	138	138
130	97.2	97.2	97.2	97.1	97.0	96.9	96.6	96.4	96.1
140	62.8	62.8	62.7	62.7	62.5	62.4	62.1	61.9	61.6
150	35.6	35.5	35.5	35.2	35.1	35.2	34.9	34.7	34.4
160	15.9	15.9	15.8	15.7	15.6	15.5	15.3	15.1	14.9
170	3.98	3.96	3.92	3.87	3.79	3.70	3.60	3.48	3.42
					$\varkappa=0.3$				
0°	924	924	924	924	924	924	924	924	924
1	810	841	864	880	890	897	902	906	908
2	798	818	837	853	866	876	884	889	894
3	792	807	822	836	849	859	868	875	881
4	789	800	812	825	836	846	855	863	869
5	785	795	805	816	826	836	845	852	859
10	770	774	780	787	793	800	806	812	818
15	750	754	758	762	767	771	776	781	785
20	728	730	733	737	740	744	747	751	755
25	703	705	707	710	712	715	718	721	724
30	676	677	679	681	683	685	688	690	692
35	647	648	649	651	653	655	657	658	660
40	616	617	618	620	621	623	624	626	627
45	585	586	587	588	589	590	591	593	594
50	553	553	554	555	556	557	558	559	560
60	487	488	488	489	489	490	491	492	492
70	422	422	422	423	423	424	424	424	425
80	357	358	358	358	359	359	359	359	360
90	296	296	297	297	297	297	297	297	297
100	238	239	239	239	239	239	239	239	239
110	186	186	186	186	186	186	186	186	186
120	138	138	138	138	138	138	138	138	138
130	97.2	97.2	97.2	97.2	97.2	97.1	97.0	96.8	96.7
140	62.8	62.8	62.8	62.7	62.7	62.6	62.5	62.4	62.2
150	35.6	35.6	35.5	35.5	35.4	35.4	35.3	35.1	35.0
160	15.9	15.9	15.9	15.8	15.8	15.7	15.6	15.5	15.4
170	3.98	3.97	3.95	3.92	3.89	3.84	3.78	3.72	3.64
					$\varkappa=0.2$				
0°	873	873	873	873	873	873	873	873	873
1	797	813	827	837	845	851	855	857	860
2	790	800	810	819	828	834	840	844	848
3	787	794	802	809	817	823	829	834	838
4	785	790	796	803	809	815	820	825	829
5	782	787	792	797	803	808	813	817	822
10	768	770	773	776	779	782	786	789	792

TABLE 4 CONTINUED

α	g								
	0.1	.2	.3	.4	.5	.6	.7	.8	.9
15°	749	751	753	755	757	759	762	764	766
20	727	728	730	731	733	735	736	738	740
25	703	703	704	705	707	708	710	711	712
30	675	676	677	678	679	680	681	682	683
35	646	647	648	648	649	650	651	652	653
40	616	616	617	618	618	619	620	621	621
45	585	585	585	586	586	587	588	588	589
50	552	553	553	554	554	554	555	556	556
60	487	487	487	488	488	488	489	489	489
70	421	422	422	422	422	422	423	423	423
80	357	357	358	358	358	358	358	358	358
90	296	296	296	296	296	296	296	297	297
100	238	238	239	239	239	239	239	239	239
110	186	186	186	186	186	186	186	186	186
120	138	138	138	138	138	138	138	138	138
130	97.1	97.2	97.2	97.2	97.2	97.1	97.1	97.0	97.0
140	62.8	62.8	62.8	62.8	62.7	62.7	62.7	62.6	62.6
150	35.6	35.6	35.6	35.6	35.6	35.5	35.5	35.4	35.4
160	15.9	15.9	15.9	15.9	15.8	15.8	15.8	15.8	15.7
170	3.98	3.98	3.97	3.96	3.95	3.93	3.91	3.88	3.85
	$\varkappa=0.1$								
0°	827	827	827	827	827	827	827	827	827
1	788	793	798	802	806	810	812	814	816
2	786	789	792	795	798	801	803	806	808
3	784	786	788	791	793	795	798	800	802
4	782	784	786	787	788	791	793	795	797
5	780	782	783	784	786	788	789	791	793
10	767	768	768	769	770	771	772	773	774
15	749	749	750	750	751	751	752	753	754
20	727	727	728	728	728	729	729	730	730
25	702	702	703	703	704	704	704	704	705
30	675	675	675	676	676	676	677	677	677
35	646	646	646	647	647	647	647	648	648
40	616	616	616	616	616	617	617	617	617
45	574	584	584	585	585	585	585	585	586
50	552	552	552	553	553	553	553	553	553
60	487	487	487	487	487	487	487	487	488
70	421	421	421	421	422	422	422	422	422
80	357	357	357	357	357	357	357	358	358
90	296	296	296	296	296	296	296	296	296
100	238	238	238	238	238	238	238	239	239
110	186	186	186	186	186	186	186	186	186
120	138	138	138	138	138	138	138	138	138
130	97.1	97.1	97.1	97.2	97.2	97.2	97.2	97.1	97.1
140	62.8	62.8	62.8	62.8	62.8	62.8	62.8	62.7	62.7
150	35.6	35.6	35.6	35.6	35.6	35.6	35.6	35.5	35.5
160	15.9	15.9	15.9	15.9	15.9	15.9	15.9	15.9	15.9
170	398	3.98	3.98	3.98	3.98	3.97	3.97	3.97	3.95

TABLE 5.

$\Delta G(\alpha\lambda\chi_1) \cdot 10^3$

α	x_1						
	—0.5	—1.0	—0.5	0.0	0.5	1.0	1.5
	$\lambda=0.1$						
0°	0.337	0.747	0.998	1.08	1.00	0.745	0.306
5	0.339	0.745	0.994	1.08	0.998	0.742	0.307
10	0.345	0.741	0.983	1.07	0.986	0.735	0.309
15	0.355	0.734	0.966	1.05	0.965	0.724	0.314
20	0.368	0.726	0.944	1.02	0.941	0.711	0.320
25	0.382	0.715	0.917	0.984	0.911	0.694	0.329
30	0.398	0.702	0.886	0.946	0.877	0.676	0.338

TABLE 5 CONTINUED

α	x_1						
	−1.5	−1.0	−0.5	0.0	0.5	1.0	1.5
35°	0.414	0.687	0.852	0.904	0.840	0.650	0.349
40	0.429	0.670	0.815	0.859	0.800	0.635	0.356
45	0.442	0.651	0.755	0.812	0.759	0.612	0.370
50	0.453	0.630	0.734	0.763	0.715	0.588	0.378
60	0.464	0.580	0.647	0.662	0.626	0.536	0.389
70	0.459	0.522	0.556	0.561	0.535	0.478	0.388
80	0.435	0.456	0.466	0.463	0.446	0.416	0.372
90	0.393	0.386	0.378	0.369	0.361	0.351	0.342
100	0.338	0.313	0.295	0.284	0.281	0.285	0.297
110	0.272	0.241	0.219	0.209	0.209	0.220	0.243
120	0.204	0.173	0.153	0.144	0.146	0.160	0.185
130	0.139	0.114	0.099	0.092	0.094	0.106	0.128
140	0.083	0.066	0.056	0.052	0.054	0.062	0.077
150	0.040	0.032	0.026	0.024	0.026	0.030	0.038
160	0.014	0.011	0.009	0.008	0.009	0.010	0.013
170	0.002	0.002	0.001	0.001	0.001	0.001	0.002
				$\lambda=0.2$			
0°	1.72	3.32	4.31	4.67	4.35	3.31	1.48
5	1.73	3.31	4.30	4.65	4.33	3.29	1.48
10	1.75	3.29	4.25	4.59	4.28	3.26	1.49
15	1.78	3.26	4.17	4.50	4.20	3.21	1.50
20	1.82	3.21	4.08	4.38	4.09	3.15	1.53
25	1.86	3.16	3.96	4.24	3.96	3.08	1.55
30	1.91	3.09	3.82	4.07	3.81	2.99	1.59
35	1.95	3.02	3.67	3.89	3.65	2.90	1.62
40	1.99	2.94	3.51	3.70	3.47	2.80	1.65
45	2.03	2.84	3.34	3.50	3.29	2.70	1.68
50	2.05	2.74	3.16	3.28	3.10	2.58	1.71
60	2.05	2.51	2.78	2.85	2.71	2.34	1.73
70	1.99	2.24	2.39	2.41	2.31	2.08	1.71
80	1.86	1.95	1.99	1.98	1.92	1.80	1.62
90	1.66	1.64	1.61	1.58	1.55	1.51	1.47
100	1.41	1.32	1.26	1.22	1.20	1.22	1.27
110	1.13	1.01	0.932	0.891	0.893	0.940	1.04
120	0.843	0.726	0.650	0.614	0.623	0.678	0.782
130	0.571	0.478	0.416	0.390	0.400	0.448	0.537
140	0.340	0.277	0.236	0.220	0.228	0.262	0.323
150	0.165	0.132	0.111	0.102	0.107	0.126	0.159
160	0.056	0.044	0.037	0.034	0.036	0.043	0.055
170	0.008	0.006	0.005	0.005	0.005	0.006	0.008
				$\lambda=0.3$			
0°	4.83	8.34	10.6	11.4	10.7	8.34	4.07
5	4.84	8.31	10.5	11.4	10.7	8.31	4.07
10	4.87	8.26	10.4	11.2	10.5	8.23	4.08
15	4.92	8.16	10.2	11.0	10.3	8.10	4.10
20	4.98	8.04	9.98	10.7	10.1	7.94	4.14
25	5.04	7.89	9.69	10.4	9.74	7.74	4.18
30	5.10	7.71	9.36	9.95	9.38	7.52	4.23
35	5.16	7.51	8.99	9.51	8.98	7.28	4.28
40	5.21	7.28	8.59	9.04	8.54	7.02	4.33
45	5.23	7.03	8.16	8.54	8.09	6.74	4.37
50	5.23	6.76	7.71	8.02	7.62	6.44	4.40
60	5.14	6.15	6.77	6.95	6.65	5.82	4.39
70	4.90	5.47	5.80	5.88	5.65	5.14	4.26
80	4.51	4.72	4.84	4.83	4.70	4.43	4.00
90	3.98	3.95	3.90	3.85	3.78	3.70	3.60
100	3.36	3.17	3.03	2.95	2.93	2.97	3.08
110	2.67	2.41	2.24	2.15	2.16	2.22	2.49
120	1.97	1.72	1.56	1.48	1.50	1.63	1.87
130	1.33	1.13	0.995	0.937	0.960	1.07	1.28
140	0.784	0.651	0.563	0.526	0.544	0.621	0.764
150	0.382	0.309	0.263	0.244	0.255	0.297	0.374
160	0.130	0.103	0.086	0.080	0.084	0.100	0.128
170	0.018	0.014	0.012	0.011	0.012	0.014	0.018

TABLE 5 CONTINUED

α	x_1						
	−1.5	−1.0	−0.5	0.0	0.5	1.0	1.5
				$\lambda=0.4$			
0°	10.06	16.7	20.7	22.2	21.1	16.8	8.93
5	10.6	16.6	20.6	22.1	21.0	16.8	8.93
10	10.6	16.5	20.3	21.9	20.7	16.6	8.93
15	10.6	16.3	20.0	21.4	20.3	16.3	8.94
20	10.7	16.0	19.5	20.9	19.8	16.0	8.96
25	10.7	15.7	18.9	20.2	19.2	15.6	9.00
30	10.8	15.3	18.3	19.4	18.4	15.1	9.03
35	10.8	14.9	17.5	18.5	17.6	14.6	9.06
40	10.8	14.4	16.7	17.6	16.8	14.1	9.10
45	10.7	13.9	15.9	16.6	15.9	13.5	9.10
50	10.6	13.3	15.0	15.6	15.0	12.9	9.08
60	10.2	12.0	13.2	13.5	13.0	11.5	8.91
70	9.59	10.6	11.2	11.4	11.1	10.1	8.52
80	8.71	9.12	9.35	9.37	9.17	8.69	7.90
90	7.61	7.58	7.52	7.45	7.35	7.21	7.03
100	6.34	6.04	5.82	5.70	5.67	5.75	5.97
110	5.00	4.58	4.29	4.15	4.17	4.37	4.78
120	3.68	3.26	2.97	2.84	2.88	3.11	3.56
130	2.46	2.12	1.89	1.79	1.83	2.03	2.41
140	1.45	1.22	1.06	1.00	1.03	1.17	1.43
150	0.701	0.576	0.495	0.462	0.480	0.557	0.696
160	0.237	0.191	0.162	0.150	0.157	0.185	0.236
170	0.034	0.027	0.022	0.021	0.022	0.026	0.034
				$\lambda=0.5$			
0°	20.2	29.6	35.9	38.6	37.0	30.3	17.5
5	20.2	29.6	35.8	38.4	36.9	30.2	17.5
10	20.2	29.3	35.4	37.9	36.4	29.9	17.5
15	20.2	28.9	34.8	37.2	35.7	29.4	17.4
20	20.1	28.4	33.9	36.2	34.8	28.8	17.4
25	20.1	27.8	32.9	35.0	33.6	28.0	17.4
30	20.0	27.0	31.7	33.6	32.4	27.2	17.3
35	19.8	26.2	30.4	32.2	31.0	26.2	17.2
40	19.6	25.3	29.0	30.5	29.4	25.2	17.1
45	19.4	24.3	27.5	28.8	27.8	24.1	17.0
50	19.0	23.2	26.0	27.1	26.2	22.9	16.8
60	18.0	20.9	22.7	23.4	22.8	20.5	16.2
70	16.7	18.3	19.3	19.7	19.3	17.9	15.2
80	15.0	15.6	16.1	16.2	15.9	15.2	13.9
90	12.9	12.9	12.9	12.8	12.7	12.5	12.2
100	10.6	10.2	9.94	9.78	9.78	9.93	10.3
110	8.33	7.72	7.30	7.10	1.16	7.49	8.16
120	6.07	5.45	5.04	4.85	4.93	5.30	6.02
130	4.04	3.53	3.19	3.04	3.11	3.44	4.05
140	2.37	2.02	1.78	1.69	1.74	1.96	2.38
150	1.14	0.948	0.825	0.773	0.804	0.924	1.15
160	0.383	0.313	0.268	0.250	0.261	0.305	0.386
170	0.054	0.044	0.037	0.034	0.036	0.042	0.054
				$\lambda=0.6$			
0°	35.8	49.3	58.6	62.8	60.9	51.4	32.3
5	35.8	49.2	58.4	62.5	60.6	51.2	32.3
10	35.7	48.7	57.7	61.7	59.8	50.6	32.1
15	35.5	48.0	56.6	60.5	58.7	49.8	32.0
20	35.3	47.1	55.3	58.9	57.1	49.7	31.7
25	35.0	46.0	53.6	57.0	55.3	47.3	31.5
30	34.6	44.7	51.7	54.8	53.2	45.8	31.2
35	34.1	43.3	49.5	52.3	50.8	44.1	30.8
40	33.5	41.6	47.2	49.6	48.3	42.3	30.4
45	32.7	39.9	44.7	46.9	45.7	40.4	29.9
50	31.9	38.0	42.2	43.9	42.9	38.3	29.2

TABLE 5 CONTINUED

α	x_1						
	−1.5	−1.0	−0.5	0.0	0.5	1.0	1.5
60°	29.8	34.0	36.8	37.8	37.2	34.0	27.7
70	27.1	29.6	31.3	32.0	31.5	29.5	25.7
80	24.0	25.1	25.8	26.1	25.9	24.9	23.1
90	20.5	20.6	20.7	20.7	20.6	20.4	20.0
100	16.7	16.2	15.9	15.7	15.8	16.0	16.6
110	12.9	12.1	11.6	11.4	11.5	12.0	13.0
120	9.36	8.52	7.96	7.72	7.84	8.40	9.49
130	6.18	5.48	5.01	4.81	4.92	5.40	6.31
140	3.60	3.11	2.79	2.65	2.73	3.06	3.68
150	1.72	1.45	1.28	1.21	1.25	1.43	1.76
160	0.574	0.476	0.412	0.386	0.402	0.465	0.584
170	0.081	0.066	0.056	0.052	0.054	0.064	0.081
				$\lambda=0.7$			
0°	60.9	79.4	92.5	99.0	97.2	84.8	58.3
5	60.8	79.2	92.2	98.6	96.8	84.5	58.2
10	60.5	78.4	91.1	97.4	95.6	83.6	57.8
15	60.1	77.3	89.5	95.5	93.8	82.1	57.3
20	59.4	75.7	78.2	92.9	91.3	80.2	56.7
25	58.6	73.8	84.6	89.9	88.3	78.0	55.9
30	57.6	71.6	81.5	86.4	84.9	75.3	55.0
35	56.4	69.1	78.0	82.0	81.1	72.4	53.9
40	55.0	66.4	74.3	78.3	77.1	69.3	52.7
45	53.4	63.4	70.4	73.8	72.8	65.9	51.3
50	51.7	60.2	66.3	69.2	68.3	62.4	49.9
60	47.5	53.5	57.6	59.7	59.1	55.0	46.4
70	42.7	46.3	48.9	50.1	49.8	47.4	42.2
80	37.2	39.0	40.2	40.9	40.8	39.7	37.3
90	31.4	31.7	32.0	32.2	32.3	32.2	31.8
100	25.3	24.8	24.4	24.4	24.6	25.1	25.9
110	19.4	18.4	17.8	17.5	17.8	18.6	20.0
120	13.9	12.8	12.1	11.8	12.1	12.9	14.4
130	9.08	8.18	7.58	7.33	7.51	8.19	9.43
140	5.23	4.60	4.18	4.01	4.13	4.59	5.47
150	2.48	2.13	1.90	1.80	1.87	2.11	2.58
160	0.821	0.691	0.606	0.570	0.592	0.680	0.845
170	0.115	0.094	0.081	0.076	0.079	0.092	0.116
				$\lambda=0.8$			
0°	103	128	146	156	156	141	107
5	103	127	146	156	155	140	106
10	102	126	144	154	153	139	106
15	101	124	141	151	150	136	104
20	99.5	121	138	147	146	133	103
25	97.6	118	133	142	141	129	101
30	95.4	114	128	136	136	125	98.6
35	92.9	110	122	130	130	120	96.0
40	90.0	105	117	123	123	114	93.1
45	86.9	100	110	116	116	108	89.9
50	83.4	95.2	104	109	109	102	86.9
60	75.6	83.9	90.1	93.7	93.9	89.6	78.8
70	66.9	72.1	76.0	78.4	78.8	76.5	70.1
80	57.5	60.1	62.3	63.7	64.2	63.4	60.7
90	47.7	48.5	49.3	50.0	50.5	50.8	50.7
100	38.0	37.5	37.4	37.6	38.1	39.1	40.6
110	28.7	27.6	27.0	26.9	27.4	28.6	30.8
120	20.3	19.0	18.2	18.0	18.4	19.6	21.8
130	13.2	12.9	11.3	11.0	11.3	12.3	14.1
140	7.47	6.68	6.17	5.97	6.15	6.78	7.99
150	3.49	3.06	2.77	2.66	2.74	3.07	3.70
160	1.14	0.980	0.871	0.827	0.856	0.973	1.20
170	0.158	0.132	0.115	0.108	0.112	0.121	0.161

TABLE 5 CONTINUED

α	x_1						
	—1.5	—1.0	—0.5	0.0	0.5	1.0	1.5
				$\lambda=0.9$			
0°	183	216	243	260	264	250	211
5	182	216	242	259	263	250	210
10	181	213	239	256	260	247	208
15	178	210	235	251	255	242	205
20	175	205	229	244	248	236	201
25	171	199	221	236	240	229	196
30	166	192	213	226	230	220	191
35	161	185	204	216	220	211	184
40	155	176	193	205	208	201	177
45	148	167	182	193	196	190	170
50	141	158	171	180	183	178	161
60	126	138	148	154	157	155	144
70	109	117	124	129	131	130	125
80	92.3	96.8	101	104	106	107	105
90	75.2	71.1	79.0	80.8	82.6	84.3	85.6
100	58.8	58.9	59.3	60.3	61.7	63.8	66.8
110	43.6	42.7	43.2	42.6	43.7	45.9	49.4
120	30.3	29.0	28.2	28.2	29.0	30.8	34.1
130	19.2	18.0	17.2	17.0	17.6	19.0	21.5
140	10.8	9.86	9.27	9.08	9.36	10.2	11.9
150	4.95	4.43	4.09	3.96	4.09	4.54	5.39
160	1.59	1.39	1.26	1.21	1.25	1.40	1.70
170	0.216	0.184	0.163	0.155	0.160	0.182	0.224
				$\lambda=0.95$			
0°	261	302	355	359	423	419	386
5	260	301	334	358	421	417	385
10	258	298	330	353	416	412	381
15	254	292	324	346	408	405	375
20	249	286	316	336	397	394	366
25	243	277	305	326	384	382	356
30	235	267	294	313	368	367	344
35	237	256	280	298	351	350	331
40	217	244	266	282	332	332	316
45	207	231	251	266	312	313	300
50	196	217	235	248	292	293	283
60	173	188	202	212	249	252	247
70	149	159	168	176	206	210	209
80	124	130	136	141	165	170	172
90	99.7	103	106	109	127	132	137
100	76.9	77.6	78.8	80.7	93.6	98.0	104
110	56.2	55.6	55.7	56.5	65.3	69.1	74.7
120	38.5	37.3	36.8	37.0	42.6	45.4	50.2
130	24.1	22.9	22.2	22.1	25.3	27.3	30.8
140	13.1	12.4	11.8	11.6	13.2	14.4	16.6
150	6.02	5.47	5.12	5.00	5.66	6.24	7.31
160	1.91	1.69	1.55	1.50	1.69	1.88	2.24
170	0.255	0.220	0.198	0.189	0.211	0.237	0.288
				$\lambda=1.0$			
0°	575	637	695	748	793	827	845
5	573	635	693	745	789	824	842
10	567	628	684	736	780	814	833
15	557	616	671	721	764	798	817
20	543	600	653	701	743	776	794
25	527	580	631	677	780	750	771
30	507	558	605	649	688	719	741
35	485	532	576	617	654	685	707
40	461	504	544	582	617	647	670
45	435	474	511	546	578	607	630
50	408	442	476	507	537	568	588
60	350	377	404	428	454	478	501
70	292	311	330	350	370	391	414

TABLE 5 CONTINUED

α	x_1						
	—1.5	—1.0	—0.5	0.0	0.5	1.0	1.5
80	235	248	261	276	291	309	330
90	182	190	198	208	220	234	252
100	135	139	144	150	158	169	184
110	95.0	96.2	98.5	102	108	116	127
120	62.3	62.1	62.8	64.7	68.1	73.5	81.9
130	37.4	36.6	36.5	37.3	39.2	42.6	48.1
140	19.7	19.0	18.6	18.9	19.8	21.6	24.8
150	8.55	8.04	7.77	7.79	8.15	8.98	10.04
160	2.60	2.38	2.26	2.24	2.34	2.60	3.06
170	0.332	0.297	0.275	0.269	0.280	0.314	0.377

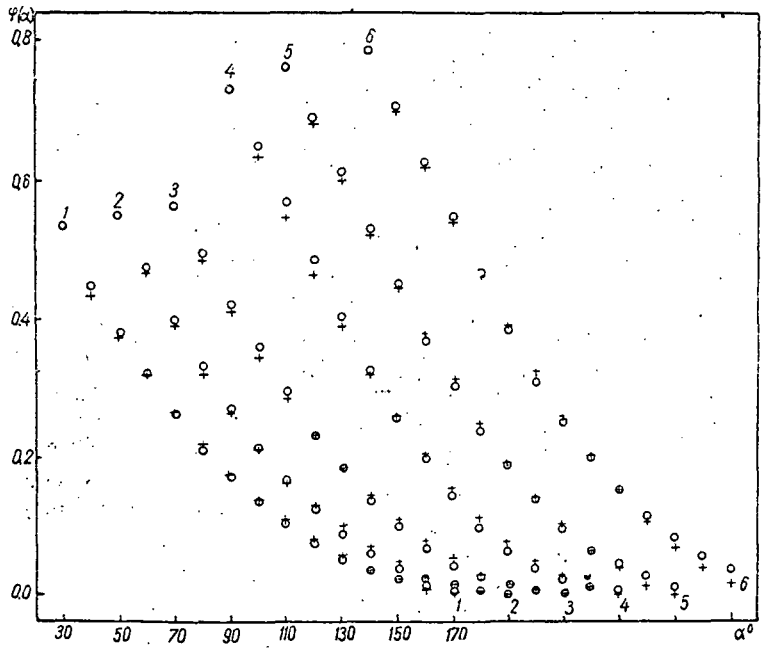

Figure 3. Integral brightness of spheres in terms of phase angle (for $\alpha \geq 30°$). The circles are the observed values, the crosses the calculated values. Each succeeding phase curve was shifted relative to the preceding one 10° with respect to α.

Experimental data show that the derivative $\frac{d\Phi}{d\alpha}\Big|\alpha = \alpha_0$ may be determined with a substantial error, so it is more convenient, initially, to find the value of λ from the disk brightness. Then, using Eq. (11), let us calculate the left side of Eq. (6) and by varying the value of the derivative $\frac{d\Phi}{d\alpha}\Big|\alpha = \alpha_0$ reach equality of both sides of this equation. It was taken that $\alpha_0 = 30°$ for the check made of Eqs. (11) and (12) in terms of sphere measurements. Figure 3 shows the observed (circles) and calculated (crosses) values of the phase function for all six spheres, and as we see, it is only for $\alpha > 150°$ that we find quite large differences between the observed and calculated phase function values for some of the spheres.

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